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A new method for CB and CBO cross-sections of 1s–2s transition in $e^- - He^+$ collision

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Among the familiar methods used for the study of electron ion collision processes, Coulomb-Born (CB) and Coulomb-Born-Oppenheimer (CBO) approximations are well known. The simple e^-–He^+ system has been investigated earlier in CB and CBO approximations by Burgess *et al* (1970), employing the method of partial wave analysis. This method leads to lengthily and uneconomic computations at high energies, while, in general the approximations like CB or CBO are more suitable at high energies. Moreover, this method is not appropriate for heavy particle impacts. Here we present a new method for the straight forward evaluation of the CB and CBO cross-sections without any partial wave analysis. This new procedure is simple and can be conveniently used for high energies with little computational effort.

In the CBO approximation the direct and exchange amplitudes f and g respectively, are given as (Seaton 1962, Mott & Massey 1965),

$$f = -\frac{1}{2\pi}\int\phi_n^*(\boldsymbol{r}_2)\chi(z-1,\boldsymbol{K}_n,\boldsymbol{r}_1)\left[\frac{1}{r_{12}}-\frac{1}{r_1}\right]\phi_1(\boldsymbol{r}_2)\chi(z-1,\boldsymbol{K}_1,\boldsymbol{r}_1)d\boldsymbol{r}_1 d\boldsymbol{r}_2 \quad (1)$$

$$g = -\frac{1}{2\pi}\int\phi_n^*(\boldsymbol{r}_1)\chi(z-1,-\boldsymbol{K}_n,\boldsymbol{r}_2)\left[\frac{1}{r_{12}}-\frac{1}{r_1}\right]\phi_1(\boldsymbol{r}_2)\chi(z-1,\boldsymbol{K}_1,\boldsymbol{r}_1)d\boldsymbol{r}_1 d\boldsymbol{r}_2 \quad \ldots (2)$$

with

$$\chi(z-1,-\boldsymbol{K}_n,\boldsymbol{r}) = \exp(\tfrac{1}{2}\pi\alpha_2)\Gamma(1-i\alpha_2)\exp(-i\boldsymbol{K}_n\cdot\boldsymbol{r})\,{}_1F_1\{i\alpha_2;1;i(K_nr+\boldsymbol{K}_n\cdot\boldsymbol{r})\} \quad \ldots (3)$$

and

$$\chi(z-1,\boldsymbol{K}_1,\boldsymbol{r}) = \exp(\tfrac{1}{2}\pi\alpha_1)\Gamma(1-i\alpha_1)\exp(i\boldsymbol{K}_1\cdot\boldsymbol{r})\,{}_1F_1\{i\alpha_1;1;i(K_1r-\boldsymbol{K}_1\cdot\boldsymbol{r})\}, \quad \ldots (4)$$

$$\alpha_2 = (z-1)/K_n;\quad \alpha_1 = (z-1)/K_1$$

Z is the nuclear charge of the target hydrogen like ion, ϕ_1 and ϕ_n are the initial and final bound state wave functions of the ion respectively. $\boldsymbol{K}_1$ and $\boldsymbol{K}_n$ are initial and final momenta of the incident electron. Here we consider $1s$–$2s$ excitation of He^+ ion. By virtue of the orthogonality of ϕ_1 and ϕ_n, the contribution due to $1/r_1$ term in the direct amplitude f vanishes. Performing the space integration over $\boldsymbol{r}_2$ in eq. (1) we get some integrals over $\boldsymbol{r}_1$, for the direct scattering amplitude f. These integrals may be generated by suitable parametric differentiation of the integral I given by

$$I = \int \exp[i(\boldsymbol{K}_1 - \boldsymbol{K}_n)\cdot\boldsymbol{r}_1]\exp(-\lambda r_1)\,{}_1F_1\{i\alpha_2, 1; i(K_n r_1 + \boldsymbol{K}_n\cdot\boldsymbol{r}_1)\} \times \\ \times {}_1F_1\{i\alpha_1, 1; i(K_1 r_1 - \boldsymbol{K}_1\cdot\boldsymbol{r}_1)\} d\boldsymbol{r}_1 \quad .. \quad (5)$$

To evaluate I the following integral representations (Nordsieck 1954) for confluent hypergeometric functions are used

$${}_1F_1\{i\alpha_j; 1, Z\} = \frac{1}{2\pi i}\oint^{(0+,1+)} dt_j\, p(\alpha_j, t_j)\exp(Zt_j) \quad .. \quad (6)$$

with

$$p(\alpha_j, t_j) = t_j^{i\alpha_j - 1}(t-1)^{-i\alpha_j} \qquad j = 1, 2$$

The integral over $\boldsymbol{r}_1$, then reduces to a two dimensional integration over t_1 and t_2. The t_2 integration is next carried out analytically by the method of residue calculation and we are left with a single dimensional integral over t_1 for the direct amplitude, which is evaluated numerically by adopting the method of Mukherjee *et al* (1975) with some modifications.

The exchange integral consists of two parts, one involving $1/r_1$ and the other, $1/r_{12}$. The first part can be integrated analytically. In the second part, first the space integrations are performed by the use of eq. (6). This leaves us with two double integrals in t_1 and t_2, the integrands of which may be obtained by parametric differentiation of the function $I\ (\lambda; q_1, \mu_1; q_2, \mu_2)$ in eq. 9 Appendix I of the work by Lewis (1956). We use integral representations for this function and its derivatives, change the order of integration and then perform the t_2 integration analytically. We are still left with a double integral which is evaluated numerically, the complex t_0 integration being done in a similar way as in the case of the direct amplitude f.

The result for the $1s$–$2s$ excitation in e^-–He^+ collision at an energy 8 Ryd is presented in this preliminary report. The total CB and CBO cross sections are respectively 13.3 and 12.1 ($\pi a_0^2 \times 10^{-3}$) in exact matching with the previous CBI and CBOI results of Burgess *et al* (1970) by the method of partial wave analysis as quoted by McDowell *et al* (1973). The differential cross-sections displayed in figure 1 show that the effect of exchange is rather prominent for backward scattering. The present method can be extended to calculate the cross sections at any energy including threshold, for an arbitary hydrogen like ion. The full results and the calculational details with be presented elsewhere

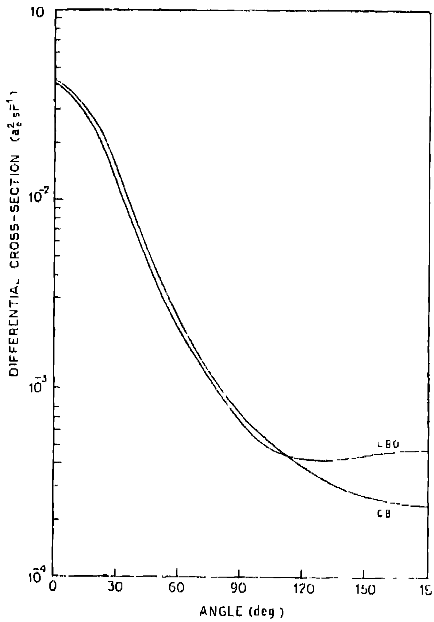

Fig. 1. Differential cross-sections for $e^- + He^+ (1s) \rightarrow e + He^+ (2s)$ at 8 Ryd.

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